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QUANTUM CAPACITANCE IN NANOSCALE TRANSISTORS

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ABSTRACT

This paper aims to introduce the third terminal of the transistor to the two terminals' device with two big contacts and a channel, to control the conductance of the channel by moving the electrochemical potential, up or down according to how much gate voltage (V_g) is needed. The gate voltage (V_g) terminals separated from the channel by an insulating material, to let no current to flow through the gate terminal, although the insulating layer is very thin no doubt there will be an undesirable leakage current to flow. The gate voltage (V_g) changes the potential inside the channel. The negative gate voltage (V_g) adds extra electrons in the channel to increase the channel energy, therefore, the entire energy levels float up. If the electrochemical potential is kept where was before applying the negative gate potential (V_g), the density of states in the channel become smaller and the channel will not conduct as well any more as before, till it reaches a point of pinch off, where the current stops due to the lack of any available states for conduction. Note that the electrochemical potential might get to the point where the channel starts to conduct through the valance band, if the gate voltage (V_g) kept increasing, but that is usually outside the voltage limit of interest.

The channel stands for one plate of a capacitor, and the gate terminal is the other plate, between them there is an insulator for the potential to form a capacitor. Therefore, another aim of this paper is to study this capacitor for low bias in the nanoscale limit to see how it looks like. The quantum capacitance (C_Q) depends on the density of the states, so it requires the wave nature of the electron. The electrostatic capacitor in origin is basically just repulsion between electrons to account for interaction between them.

KEYWORDS: Conductance, the electrochemical potential, density of states, pinch off, available states, the valance band, conduction band, Fermi- function.

1. INTRODUCTION

The nano transistor is the most important electronic device, the transistor is like a resistor in the sense that when a voltage (V_D), is applied between the source and drain terminals, current flows. In this paper the factors that affects this resistor are discussed.

The transistor has three terminals, the third one is to control the resistance, what will be shown here schematically is the third terminal (the gate) insulated from the channel by an insulating material to add another voltage (V_g). The purpose of these two voltages is completely different, (V_D) causes current to flow through the channel, ideally (V_g) causes no current flow from the gate, because of the insulation, and it sole purpose is to control the current flow through the channel.

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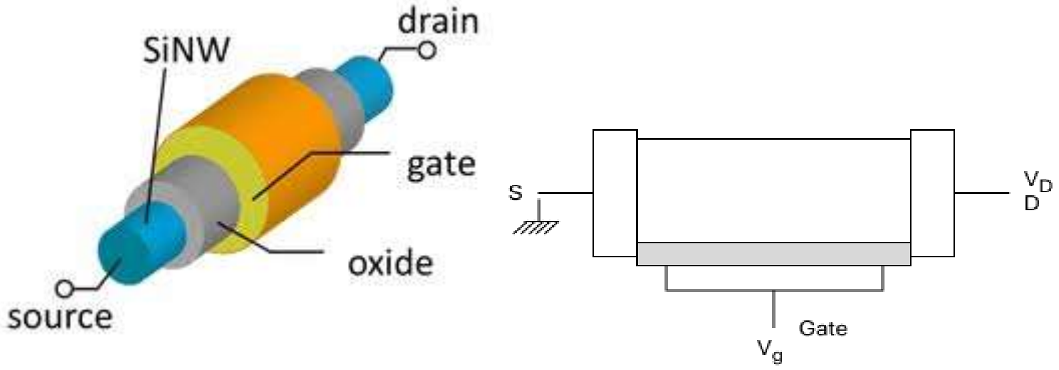
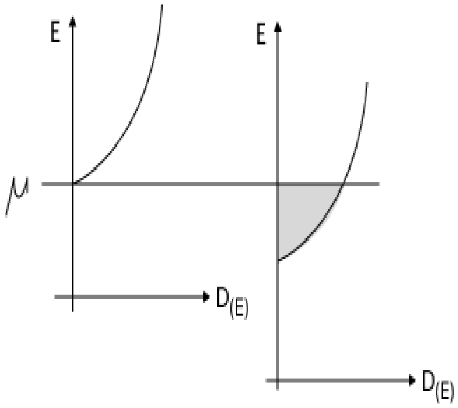


The current flow depends on the availability of energy states in the schematic diagram. (V_g) moves the energy states in the channel up or down to change the resistance of the channel, hence the energy inside the channel changes by an amount;

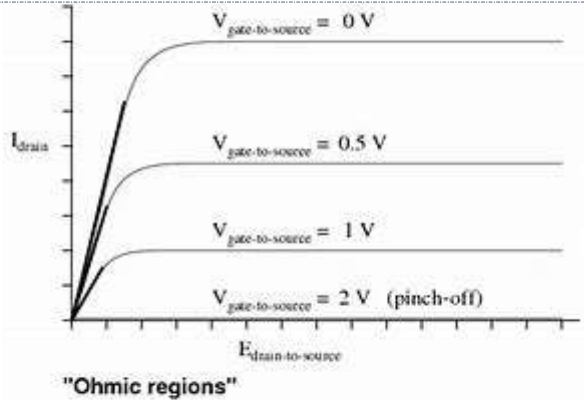
$$U = \beta(-qV_g) \quad (1)$$

Where, (q) is the electron charge.
(V_g) is the gate voltage.
(β) is a factor, the best it could be equal to (one), but usually somewhat less.

Negative potential $(-V_g)$ moves all the states up, and the positive potential $(+V_g)$ moves all of them down. So (V_g) turns the channel on for positive potential $(+V_g)$, and off for negative potential $(-V_g)$. When the electrochemical potential is pushed down below the bottom of the conduction band, or If the band of states is kept moving up till the electrochemical potential goes below the bottom of the conduction band the current flow stops. In other words, if the band of states is pushed above the electrochemical potential, no current flows in the channel.



All the equations which is going to be applied are valid only in the linear part of the low bias characteristics of the transistor.



If the channel is thought to be one plate of a capacitor, and the gate is the other plate, between them an insulator, the potential charge will be;

$$V_0 = \frac{q}{C_E} \quad (2)$$

Where, (C_E) is the electrostatic capacitor.
For a single electron, an electrostatic energy repulsion between the electrons will be;

$$U_0 = \frac{q^2}{C_E} \quad (3)$$

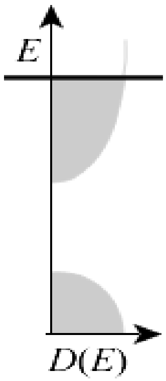
Equation (3) is nothing more than the electrostatic interaction of the nineteenth century physics, although it has been corrected now in the basis of quantum theory, but still considered as an electrostatic interaction
In quantum basis, there is another quantity;

$$(D_0) = C_Q/q^2$$

Where, (D_0) is known as the quantum capacitance, because it has the dimensions of the capacitor when multiplied by the square of the electron charge, and at the same time (C_Q) depends on the average density of states (D_0), to acquire the wave nature of the electron.

2. THE MATHEMATICAL MODEL

How to model the above description, the Fermi-Function must be introduced to determine how much states are occupied, or not. Suppose there is a two-terminal device in which the density of states in its channel is up to the electrochemical potential (μ) to tell the level up to which the states are filled, as shown in the figure below;



We are only concerned with the conduction band at the top of the diagram, and the valance band can be ignored. Controlling the conduction through the channel gives the essence of the transistor, so for this purpose the third terminal of the transistor is needed. Before connecting the gate voltage (V_g) the channel conducts well, because here are a lot of states in the channel, after the addition of the gate voltage (V_g), the states float up, and there will small number of states, and the channel does not conduct as well.

The mathematical form of the Fermi-Function is;

$$f_0 = \frac{1}{1+e^x} \quad (4)$$

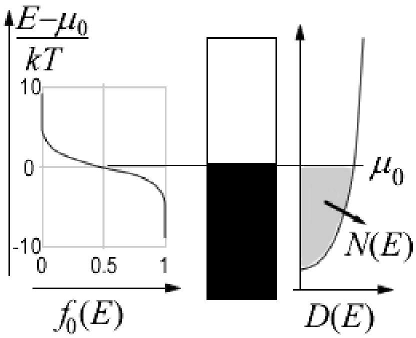
Where;

$$x = \frac{(E-\mu)}{KT}$$

At zero temperature everything below the electrochemical potential (μ) is filled, and at nonzero temperature the Fermi Function should be introduced to describe the levels' occupation, the number of electrons is given by;

$$N_0 = \int_{-\infty}^{\infty} D(E) f_0(E) dE \quad (5)$$

Where, $D(E)$ tells how many states therein.
$f_0(E)$ tells what fraction of them are filled.



When the gate potential is changed, it changes the energy potential inside the channel, as a result of the negative potential $(-V_g)$, a positive energy (U), is added to the energy of the channel, hence the density of states float up. Mathematically this can be expressed by the following equation to determine the new number of electrons after the addition of the gate voltage $(-V_g)$;

$$N = \int_{-\infty}^{\infty} D(E-U) f_0(E) dE \quad (6)$$

By transformation of variables;

$$N = \int_{-\infty}^{\infty} D(E) f_0(E+U) dE \quad (7)$$

1. **1.** Mathematical analysis and results:

Two cases can be discussed;

1. The electrochemical potential μ is below the band;
   By nondegenerative Boltzmann's approximation;

$$\frac{1}{(1+e^x)} = e^{-x} \quad (8)$$

Then, the Fermi Function can be replaced by a simple expression $\left(e^{-\left(\frac{E+U-\mu}{KT}\right)}\right)$, therefore;

$$N = \int_{-\infty}^{\infty} D(E) \left(e^{-\left(\frac{E+U-\mu}{KT}\right)}\right) dE \quad (9)$$

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Pull the exponential, $\left(e^{-\left(\frac{U}{KT}\right)}\right)$ out of the integral, because it doesn't depend on the energy anymore;

$$N = e^{-\left(\frac{U}{KT}\right)} \int_{-\infty}^{\infty} D(E) \left(e^{-\left(\frac{E-\mu}{KT}\right)}\right) dE \quad (10)$$

Whatever left inside the integral is (N_0), represented by non-degenerative Boltzmann's approximation, then;

$$N = N_0 e^{-\left(\frac{U}{KT}\right)} \quad (11)$$

Take the natural log both sides;

$$Log_e\left(\frac{N}{N_0}\right) = -\left(\frac{\mathrm{U}}{\mathrm{KT}}\right) \quad (12)$$

To determine how much change of energy inside the channel will be due to the change in (V_g), recall equation (1)

$$Log_e\left(\frac{N}{N_0}\right) = -\left(\frac{\mathrm{U}}{\mathrm{KT}}\right) = \frac{+\beta(qV_g)}{KT}$$

$$\Delta\left(Log_e\left(\frac{N}{N_0}\right)\right) = \left(\frac{\beta q}{KT}\right) \Delta V_g$$

Turn the elation around;

$$\Delta(V_g) = \left(\frac{KT}{q\beta}\right) \Delta\left(Log_e\left(\frac{N}{N_0}\right)\right) \quad (13)$$

Where, $\left(\frac{KT}{q\beta}\right) = 25mV$ at room temperature.

When the electron density is needed to change by a decade (a factor of ten), then (60mV) change in (V_g) is required. Since $Log_e\left(\frac{N}{N_0}\right) = 2.3$, ideally the best change expected in (V_g) is (60mV) per decade.

By measurement if the change in (V_g) happened to be greater than (60 mV) per decade, the work should have to be in β to make it more bigger to approach unity, in the opposite if (V_g) is changed by less than (60mV), that means (β) is greater than one, immediately gets everybody to pay attention, either a big mistake has taken place or something valuable has been discovered.

2. The electrochemical potential μ is inside the band of states;

When Fermi Level is inside the conduction band of states, where there are enough number of electrons, the non-degenerative approximation of equation (9) should not be used anymore, an alternative modified electron density of equation (6) will be used. Take the derivative of equation (6) to get;

$$\frac{dN}{dU} = \int_{-\infty}^{\infty} D(E) \frac{\partial f_0(E+U)}{\partial E} dE = -D_0 (14)$$

(D_0) is a negative quantity, because the derivative of the Fermi- function is negative, so the energy goes up and the derivative goes down in the energy range of interest.

Because of those extra electrons in the channel, the change in the charge, $Q = d(qN)$, due to the change in potential, $d\left(\frac{U}{-q}\right)$ in the channel, is given by;

$$\frac{d(qN)}{d\left(\frac{U}{-q}\right)} = q^2 D_0 = C_Q \quad (15)$$

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Equation (15) above dimensionally like a capacitor, and because of the differentiation $\left(\frac{dQ}{d(V)}\right)$, that why it is called a quantum capacitor (C_Q). That quantum capacitance (C_Q) depends on the density of states $D(E)$, and related to the average density of states $(-D_0)$, so it acquires the wave nature of an electron.
From equation (15);

$$-q^2\frac{d(N)}{d(U)} = q^2 D_0 = C_Q \implies D_0 = \frac{C_Q}{q^2} \quad (16)$$

Recall the excess energy inside the channel;

$$U = \beta(-qV_g)$$

Then;

$$\frac{dU}{d(-qV_g)} = \beta (17)$$

This only true if the charging energy due electrons is ignored, but when there are a lot of electrons, an extra term must be added to compensate for interaction of each electron with the rest of other ones.

$$U = \beta(-qV_g) + U_0(N - N_0) \qquad (18)$$

Where, N_0 is the number of electrons in the neutral state.
N is the number of electrons increased to the value N.
U_0 is the (one) electron charging energy.

Change in the number of electrons give rise to negative energy $\beta(-qV_g)$. (U_0) tells how much the potential would change due to one electron in the channel, and the actual total change will be given by (U_0) times the change in the number of elections $(N - N_0)$. When $(N = N_0)$, the channel is neutral and all the electrons compensate positive charge of protons in the channel, but when extra electrons came in, give rise to negative energy (out of equilibrium case).

Treating the electrons as uncharged particles. As the electrochemical potential (μ) is lowered, the number of electrons increases, to lower (μ) further the number of electrons still goes up, hence the overall potential energy increases. But practically electrons are negatively charged particles, that is why what will be seen is a different picture other than what is really expected, and any time extra electrons came in, will make it harder for the other electrons to come into the channel, at the same time these extra electrons make the energy states to float, so the states goes down less than what is expected.

Back to equation (18);

$$U = \beta(-qV_g) + U_0(N - N_0) \qquad (18)$$

By differentiation;

$$\frac{dU}{d(-qV_g)} = \beta + U_0\frac{dN}{d(-qV_g)}$$

Using the chain rule;

$$\frac{dU}{d(-qV_g)} = \beta + U_0\left(\frac{dN}{dU}\right)\left(\frac{dU}{d(-qV_g)}\right)$$

$$\frac{dU}{d(-qV_g)} = \beta - U_0 D_0\left(\frac{dU}{d(-qV_g)}\right)$$

Where, $\left(\frac{dN}{dU}\right) = -D_0$.

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The quantity $\left(\frac{dU}{d(-qV_g)}\right)$ is in both sides of the equation, move them all to the left side;

$$\frac{dU}{d(-qV_g)} + U_0 D_0 \left(\frac{dU}{d(-qV_g)}\right) = \beta$$

$$\frac{dU}{d(-qV_g)} = \frac{\beta}{1 + U_0 D_0} \qquad (19)$$

The change in potential energy due to the change in the gate voltage is given by none ideal factor (β), in equation (19) the change in potential has been shown less than (β), depending on how much big $(U_0 D_0)$ is, where (D_0) is the is the average density of states at equilibrium and (U_0) is the single electron charging energy. For small values of (D_0), the term $(U_0 D_0)$ can be ignored and the change in potential energy due to the change in the gate voltage is equal to (β). Generally, this extra term should be included.

Another way to write this term $(U_0 D_0)$ is by introducing the electrostatic capacitance (C_E), to distinguish it from the quantum capacitance (C_Q). The electrostatic capacitance is just the repulsion energy (U_0) between electrons, when the interaction between the electrons is considered. In contrast (D_0) represents the quantum capacitance, which depends on the average density of states (D_0) to acquire the wave nature of the electron.
Since the quantum capacitance is given by equating (16);

$$D_0 = \frac{C_Q}{q^2}$$

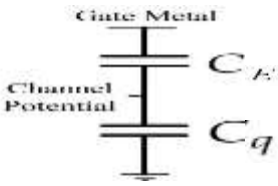
And the electrostatic capacitance is given by;

$$U_0 = \frac{q^2}{C_E}$$

Then;

$$\frac{dU}{d(-qV_g)} = \frac{\beta}{1 + U_0 D_0} = \frac{\beta}{1 + \frac{C_Q}{q^2} \cdot \frac{q^2}{C_E}} = \frac{\beta C_E}{C_E + C_Q} \qquad (20)$$

Equation (20) approximately can be visualized as an electric circuit of capacitors in series which can be represented as follows;



3. CONCLUSION AND COMMENTS

The result arrived to in equation (20), the gate voltage (βV_g) is connected to (C_E) terminal and (C_Q) is connected to the ground at the end terminal of the source, from the middle node the potential of the channel is taken. When the density of states (D_0) is low, the none degenerative case is applied, and the Fermi energy is down in the band gap. If (D_0) is small, then (C_Q) is also small to act like very high resistor, and the entire gate voltage (βV_g) essentially appears a cross the capacitor (C_E) at the middle node of the channel.

In contrast when the Fermi level energy (μ) is inside the conduction band the average density of states (D_0) is very high, hence (C_Q) is large to act like a very small resistor and potential in the middle node (the channel), is very much less than the applied voltage at the gate (βV_g).

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The basic framework, in terms of which to understand how the gate voltage changes the channel potential energy by making the conduction band to float up or to sink down to control the conductance, which is the essential physics underlining the operation of the field effect transistors.

The goal for this paper is to demonstrate that, the essential operating principles of nano transistors are much different from those transistors which had been described some decades ago, but these operating principles are remarkably simple and easy to understand. The approach is based on a new understanding of electron transport that has emerged from research on molecular and nanoscale electronics [1], but it retains much of the original theory of the MOSFET.

Capacitors are critical components in nano scale electronics, nano scale capacitors show strong size effects both quantitatively and qualitatively, quantum effects conspire to violate what is expected from classical electrostatics. Careful re-examination of many body quantum mechanics at nano scale reveals that the overall capacitance can be either smaller or larger than what is expected from electrostatics, depending on the size of the quantum capacitance.

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